Journal of Financial Management Strategy
Alzahra University
Vol. 11, No. 43, Winter 2023

pp. 11-12

Research Paper

Generalized Black-Scholes Model under Garch Volatility with Conditional value-at-risk Calculation in Derivative Pricing[1]

Hossein Nasrollahi[2], Mohammadreza Haddadi[3], Manizheh Goudarzi[4]

Received: 2023/05/21 Accepted: 2023/11/15

INTRODUCTION

The capital market plays a crucial role in the economic growth and progress of any country. Given the inherent high risks associated with investments in this market, the development of tools to mitigate risk is essential. Option contracts have emerged as valuable tools in risk management, and their pricing is a critical aspect. The Black-Scholes model is widely used for pricing various option contracts, relying on factors such as the strike price, asset base price, expected volatility, time to maturity, and risk-free interest rate. However, a significant limitation of this model is the assumption of constant returns volatility. This research aims to enhance the accuracy of forecasting European call option prices by addressing the limitation of the constant volatility assumption and replacing it with time series models. Specifically, the research explores the extension of the Black-Scholes model under GARCH (Generalized Autoregressive Conditional Heteroskedasticity) volatilities to capture the stochastic nature of volatilities and provide a more realistic representation of option prices.

MATERIALS AND METHODS

This research adopts an analytical-applied approach, utilizing data on Iran Khodro's share prices from 9/1/1399 to 9/23/1401. The analysis is conducted using R software version 1-3-4. The methodology involves calculating volatility through standard GARCH, threshold GARCH, exponential GARCH, and historical data. Subsequently, the Black-Scholes model

1. DOI: 10.22051/JFM.2023.43857.2828

2. M.Sc. Student, Department of Financial Mathematics, Ayatollah Borujerdi University, Borujerd, Iran. Email: Nasrollahi_hossein@yahoo.com.

3. Assistant Professor, Department of Financial Mathematics, Faculty of Basic Sciences, Ayatollah Borujerdi University, Borujerd, Iran. Corresponding Author. Email: haddadi@abru.ac.ir.

4. Assistant Professor, Department of Statistics, Faculty of Basic Sciences, Ayatollah Borujerdi University, Borujerd, Iran. Email: m.goudarzi@abru.ac.ir.

is extended under these stochastic volatilities. The research calculates call option prices using this extended model and compares them across different volatilities.

An innovative aspect of this article lies in the performance analysis of pricing models over short-term, medium-term, and long-term periods, incorporating conditional value-at-risk for each price under analytical and simulation models. Additionally, the research aims to utilize the conditional value-at-risk criterion to analyze the pricing performance of these models.

FINDINGS

The research presents calculated prices and errors in three distinct periods: short-term, medium-term, and long-term, considering three different prices. The analysis is conducted through two methods: analytical solution and Monte Carlo simulation. A comparative assessment of the four volatilities reveals that the Monte Carlo method exhibits a lower error. Additionally, the research indicates that pricing errors are lower in the short term compared to the long term, irrespective of the analytical or Monte Carlo methods. Furthermore, the study finds that the values of value at risk closely align with conditional value at risk. In the short and medium term, exponential GARCH demonstrates the highest conditional value at risk, whereas standard GARCH achieves the highest in the long term. Notably, historical volatility consistently produces the lowest conditional value at risk across all time periods, suggesting that its use in option pricing yields a more realistic risk profile. To validate the results, call option prices were calculated using the Monte Carlo method in all scenarios, confirming the obtained results.

CONCLUSION

The Black-Scholes model, commonly employed for pricing various options contracts, relies on the assumption of constant return fluctuation. This research aims to predict and compare the performance of Black-Scholes models in call option pricing, employing different volatility approaches, including historical data and GARCH models. Given the pivotal role of volatility in option pricing, historical volatility, standard GARCH, exponential GARCH, and threshold GARCH were utilized to identify the optimal volatility approach based on monetary value and time to maturity. The research further evaluated the pricing performance of the models across short-term, medium-term, and long-term periods, concluding by determining the conditional value at risk for each of the prices. The results indicate that, while the option price under the Black-Scholes model aligns more closely with the market price than GARCH models with historical volatilities, it also offers a more realistic risk assessment, making it a recommended approach for call option pricing.

Keywords: GARCH Volatility, Option Pricing, Black-Scholes Model, TGARCH.

JEL Classification: C13, C58, G17.